Mathematical analysis of entropy generation in couple stress fluid flow through porous channel

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Abstract: This paper studies the entropy generation in couple stress fluid flow through porous channel with fluid slippage. The solution of the non linear boundary value problem solved analytically by using Homotopy analytical method. Approximate analytical solution for velocity profile, temperature profile, entropy generation and Bejan number are obtained and investigated graphically. Also the effect of thermophysical parameters with respect to entropy generation and irreversibility ratio are analyzed graphically.

Keywords: Entropy, Couple stress, Boundary value problem, Irreversibility ratio, Homotopy analysis method.

1. INTRODUCTION

The rapid depletion of energy resources world-wide has prompted almost every country in the world to focus attention on energy conservation and improving existing energy systems to minimize the energy waste. The scientific community has responded to the challenge by developing new techniques of analysis and design so that the available work destruction is either eliminated or minimized. The improvement of thermal systems has gained a growing interest due to the relations with the problems of material processing, energy conversion and environmental effects. Efficient energy utilization during the convection in any fluid flow is one of the fundamental problems of the engineering processes to improve the system. One of the methods used for the prediction of performance of the engineering processes has been the second law analysis. The second law of the thermodynamics is applied to investigate the irreversibility in terms of the generation of entropy.

Since entropy generation is the measure of the destruction of available work of the system, the determination of the active factors motivating the entropy generation is important in upgrading the system performances. Rapid progress in science and technology has led to the development of an increasing number of flow devices that involve the manipulation of fluid flow in various geometries. Many text books of fluid dynamics fails to mention that the no-slip condition remains an assumption due to unusual agreement with experimental results for a century. Nevertheless, another approach supposed that fluid can slide over a solid surface because the experimental fact was not always accepted in the past. Bejan [4-6] proposed Entropy Generation Minimization which includes general boundary conditions. He also investigated Second-law analysis in heat transfer and thermal design. The phenomenon of slip occurrence has been demonstrated by the recent theoretical and experimental studies such as Sahraoui et al. [7]. Adesanya, S.O. et al [8] presented steady magnetohydrodynamic visco-elastic heat generating/absorbing slip flow with thermal radiation through a porous medium. Interested readers can see more interesting work on the influence of magnetic field on entropy generation rate in [10-16]. Ananthaswamy et al [17] analyzed exact solution of convective heating on entropy generation rate.

The Homotopy analysis method HAM is in essence based on the extremely large freedom of constructing a Homotopy of equations. It is such kind of unbelievable freedom, combined with more and more powerful computer algebra systems, that makes the HAM pretty general and valid for problems with strong nonlinearity. The HAM greatly differs from other traditional analytic methods. In this project the Homotopy analysis method was employed to solve some nonlinear problem the result reveals that the proposed method is effective. S. J. Liao[19] proposed Homotopy analysis technique for the solution of non linear problems. This study essentially extends the earlier work of Adesanya.et.al., [18] to Entropy generation in couple stress fluid flow through porous channel with fluid slippage. The solution of the non-linear boundary value problem solved analytically by using Homotopy analytical method.

Approximate analytical solution for velocity profile, temperature profile, entropy generation and Bejan number are obtained and investigated graphically. Also the effect of thermophysical parameters with respect to entropy generation and irreversibility ratio are analyzed graphically.

2. Mathematical formulation of the problem

We consider the laminar flow of incompressible couple stress fluid bounded by a uniform porous channel of distance *d* apart. Choose the cartesian coordinate system in such a way that the *x*'-axis is taken along the infinite plate while y'-axis is normal to it as shown in Figure 1.

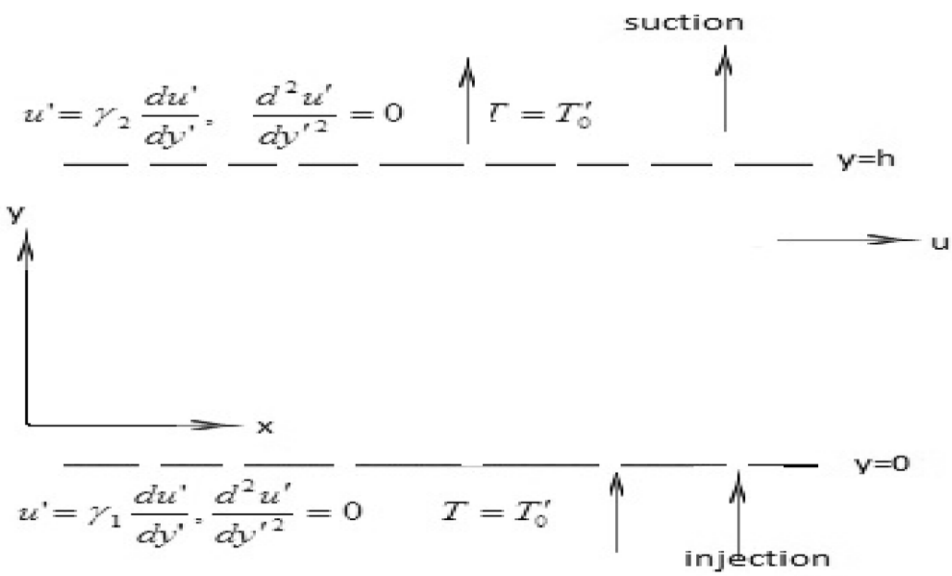

Fig.1: Schematic diagram

The flow is induced by a constant pressure gradient in the direction of the flow, the fluid is assumed to be injected into the channel with constant velocity on one part of the plate and sucked off with the same velocity on the other plate. Owing to porosity of the plates, Navier slip condition is imposed at the two plates, the equations governing the fluid flow can be written as ;

$$\rho v_o \frac{du'}{dy'} = -\frac{dP'}{dx'} + \mu \frac{d^2u'}{dy^{2'}} - \eta \frac{d^4u'}{dy^{4'}} \tag{1}$$

$$\rho C_p \frac{dT'}{dy'} = k\frac{d^2T'}{dy^{2'}} + \mu\left(\frac{du'}{dy'}\right)^2 + \eta\left(\frac{d^2u'}{dy^{2'}}\right)^2 \tag{2}$$

$$E_G = \frac{k}{T_o^2}\left(\frac{dT'}{dy'}\right)^2 + \frac{\mu}{T_o}\left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{T_o}\left(\frac{d^2u'}{dy^{2'}}\right)^2 \tag{3}$$

Additional terms in eqns. (1) – (3) are due to the couple stress effect given by Srinivasacharya et.al. [3] arising due to particle size. The appropriate boundary conditions for the uniform wall temperature with Navier slips are

$$u' = \gamma_1 \frac{du'}{dy'}, \ \frac{d^2u'}{dy^{2'}} = 0 \qquad T = T_o' \quad on \quad y = 0 \tag{4}$$

$$u' = \gamma_2 \frac{du'}{dy'}, \ \frac{d^2u'}{dy'^2} = 0 \qquad T = T_o' \ \ on \ \ y = h \tag{5}$$

By introducing the following dimensionless variables:

$$\left.\begin{aligned} & y = \frac{y'}{h},\ u = \frac{u'}{v_o},\ \theta = \frac{T' - T_o}{T_f - T_o},\ s = \frac{v_o h}{v},\ G = -\frac{h^2}{\mu v_o}\frac{dP}{dx}, \\ & a^2 = \frac{\mu h^2}{\mu},\ \Pr = \frac{v\rho C_p}{k},\ Br = \frac{v_o^2 \mu}{k\left(T_f - T_o\right)},\ \beta_1 = \frac{\gamma_1}{h},\ \beta_2 = \frac{\gamma_2}{h}, \\ & N_s = \frac{T_o^2 h^2 E_G}{k\left(T_f - T_o\right)^2},\ \Omega = \frac{T_f - T_o}{T_o} \end{aligned}\right\}$$

(6)

we have, the following dimensionless equations:

$$s\frac{du}{dy} = G + \frac{d^2u}{dy^2} - \frac{1}{a^2}\frac{d^4u}{dy^4} \tag{7}$$

$$s\Pr\frac{d\theta}{dy} = \frac{d^2\theta}{dy^2} + Br\left(\frac{du}{dy}\right)^2 + \frac{Br}{a^2}\left(\frac{d^2u}{dy^2}\right)^2 \tag{8}$$

$$N_s = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 + \frac{Br}{\Omega a^2}\left(\frac{d^2u}{dy^2}\right)^2 \tag{9}$$

together with appropriate boundary conditions

$$u = \beta_1\frac{du}{dy},\ \frac{d^2u}{dy^2} = 0,\quad \theta = 0\ \ on\ y = 0 \tag{10}$$

$$u = \beta_2\frac{du}{dy},\ \frac{d^2u}{dy^2} = 0,\quad \theta = 0\ \ on\ y = 1 \tag{11}$$

It is important to remark here that in the asymptotic case as $a \to \infty$, the work of Egunjobi and Makinde is fully recovered. Hence, their work done has been extended to the non-Newtonian case. Similarly, as $a \to \infty$, $G \to 0$ and β_1, $\beta_2 \to 0$ recover the work of Ajibade et al.

3. Approximate analytical expressions of the non-linear boundary value problem using the Homotopy analysis method

Homotopy analysis method (HAM) is a non-perturbative analytical method for obtaining series solutions to nonlinear equations and has been successfully applied to numerous problems in science and engineering. The Homotopy analysis method (HAM) is an analytic approximation method for highly nonlinear problems, proposed by the Liao in 1992 [19]. Unlike perturbation techniques, the HAM is independent of any small/large physical parameters at all. One can always transfer a nonlinear problem into an infinite number of linear sub problems by means of the HAM. Secondly, different from all of other analytic techniques, the HAM provides us a convenient way to guarantee the convergence of solution series so that it is valid even if nonlinearity becomes rather strong. In comparison with other perturbative and non-perturbative analytical methods, HAM offers the ability to adjust and control the convergence of a solution via the so-called convergence-control parameter. Besides, based on the homotopy in topology, it provides us extremely large freedom to choose equation type of linear sub-problems, base function of solution, initial guess and so on. So that complicated non-linear ODEs and PDEs can often be solved in a simple way. HAM has proved to be the most effective method for obtaining analytical solutions to highly non-linear differential equations. An approximate solution technique which does not depend upon small parameters investigated by Liao in 1995[20]. An explicit totally analytic approximation of Basis viscous flow problems analyzed by Liao in 1999[22]. As seen in our problem non-linearity dimensionless governing equations present in the laminar flow of incompressible couple stress fluid bounded by a uniform porous channel and thus, poses a greater challenge with respect to finding approximate solutions analytically. Our results show that even in this case, HAM yields excellent results. The approximate analytic expressions of the eqns. (7)-(11) by using HAM is as follows:

$$\begin{aligned} u(y) &= (A_1 - hA_5)y + (A_2 - hA_6) + \left(A_3\left(1+\frac{S\,h\,y}{2}\right) - hA_7\right)e^{ay} \\ &+ \left(A_4\left(1+\frac{S\,h\,y}{2}\right) - hA_8\right)e^{-ay} - \frac{G+h\,S\,A_1}{2}y^2 + h\frac{S\,G\,y^3}{6} \end{aligned} \tag{12}$$

$$\begin{aligned} \theta(y) &= C_1 + C_2\,e^{S\Pr y} + Br\left(A_1^2 + \frac{G^2}{a^2}\right)\frac{y}{S\Pr} - \frac{Br\,a\,A_3{}^2 e^{2ay}}{2a - S\Pr} - \frac{Br\,a\,A_4{}^2 e^{-2ay}}{2a + S\Pr} \\ &- \frac{2Br\,A_3(A_1 a + G)e^{ay}}{a^2 - S\Pr a} - \frac{2BrA_4(A_1 a + G)e^{-ay}}{a^2 + S\Pr a} + \frac{Br\,G^2}{S\Pr}\left(\frac{y^3}{3} + \frac{y^2}{S\Pr} + \frac{2y}{S^2\Pr^2}\right) \\ &- \frac{2Br\,a\,A_4 G e^{-ay}}{a^2 + S\Pr a}\left(y + \frac{2a + S\Pr}{a^2 + S\Pr a}\right) + \frac{2Br\,a\,A_3 G e^{ay}}{a^2 - S\Pr a}\left(y - \frac{2a - S\Pr}{a^2 - S\Pr a}\right) \\ &- \frac{2BrA_1 G}{S\Pr}\left(\frac{y^2}{2} + \frac{y}{S\Pr}\right) \end{aligned} \tag{13}$$

The approximate analytical expression of the Entropy generation number using the eqns. (9), (12) and (13) is given by

$$\begin{aligned} N_S &= \frac{Br}{\Omega}\begin{bmatrix} A_1 - hA_5 + \left((a + \frac{Sh}{2}(ay+1))A_3 - hA_7\right)e^{ay} \\ + \left((\frac{Sh}{2}(1-ay) - a)A_4 - hA_8\right)e^{-ay} - (G + hSA_1)y + \frac{SGh\,y^2}{2} \end{bmatrix}^2 \\ &+ \frac{Br}{\Omega a^2}\begin{bmatrix} \left((a + Sh(\frac{ay}{2}+1))A_3 - hA_7\right)a\,e^{ay} - \left((Sh(1-\frac{ay}{2}) - a)A_4 - hA_8\right)a\,e^{-ay} \\ -(G + hSA_1) + S\,G\,h\,y \end{bmatrix} \end{aligned} \tag{14}$$

Where

$$A_1 = \frac{1}{\beta_1 - \beta_2 + 1}\left[\left((1-\beta_1 a) - (1-\beta_2 a)e^a\right)A_3 + \left((1+\beta_1 a) - (1+\beta_2 a)e^{-a}\right)A_4 + \left(\frac{1}{2} - \beta_2\right)G\right] \tag{15}$$

$$A_2 = \beta_1 A_1 + (\beta_1 a - 1)A_3 - (\beta_1 a + 1)A_4 \tag{16}$$

$$A_3 = \frac{G(1 - e^{-a})}{a^2(e^a - e^{-a})} \tag{17}$$

$$A_4 = \frac{G(e^a - 1)}{a^2(e^a - e^{-a})} \tag{18}$$

$$A_5=\frac{1}{\beta_1-\beta_2+1}\begin{bmatrix}\left(e^a\left(\beta_2\,a-1\right)-\left(\beta_1\,a+1\right)\right)A_7+\left(\left(\beta_1\,a+1\right)-e^{-a}\left(\beta_2\,a+1\right)\right)A_8\\-\frac{S}{2}\left(e^a\left(\beta_2\left(a+1\right)-1\right)-1\right)A_3-\frac{S}{2}\left(e^{-a}\left(\beta_2\left(1-a\right)-1\right)-1\right)A_4\\+S\,A_1\left(\beta_2-\frac{1}{2}\right)-S\,G\left(\beta_2+\frac{1}{3}\right)\end{bmatrix} \quad (19)$$

$$A_6=\beta_1\,A_5+\left(\beta_1 a-1\right)A_7-\left(\beta_1 a+1\right)A_8-\frac{S\left(A_3+A_4\right)}{2} \quad (20)$$

$$A_7=-\frac{S}{a}\left(\frac{A_1}{a}+A_3+A_4\right)-A_8 \quad (21)$$

$$A_8=\frac{1}{a^2\left(e^{-a}-e^a\right)}\left[S\,G-S\,A_1\left(1-e^a\right)+\frac{S}{2}A_3 a\,e^a\,(a+4)+\frac{S}{2}A_4 a\left(a\,e^{-a}-2\left(e^a+e^{-a}\right)\right)\right] \quad (22)$$

$$C_2=\frac{1}{(1-e^{S\,\Pr})}\begin{pmatrix}\left(A_1^2+\frac{G^2}{a^2}\right)\frac{Br}{S\,\Pr}+\frac{Br\,a\,{A_3}^2\left(1-e^{2a}\right)}{2a-S\,\Pr}+\frac{Br\,a\,{A_4}^2\left(1-e^{-2a}\right)}{2a+S\,\Pr}\\+\frac{2Br\,A_3(A_1\,a+G)\left(1-e^a\right)}{a^2-S\,\Pr a}+\frac{2Br\,A_4(A_1\,a+G)\left(1-e^{-a}\right)}{a^2+S\,\Pr a}\\+\frac{Br\,G^2}{S\,\Pr}\left(\frac{1}{3}+\frac{1}{S\,\Pr}+\frac{2}{S^2\,\Pr^2}\right)+\frac{2Br\,a\,A_4 G}{a^2+S\,\Pr a}\left(\left(1-e^{-a}\right)\frac{2a+S\,\Pr}{a^2+S\,\Pr a}-e^{-a}\right)\\+\frac{2Br\,a\,A_3 G e^{a\,x}}{a^2-S\,\Pr a}\left(\left(1-e^a\right)\frac{2a-S\,\Pr}{a^2-S\,\Pr a}+e^a\right)-\frac{2BrA_1\,G}{S\,\Pr}\left(\frac{1}{2}+\frac{1}{S\,\Pr}\right)\end{pmatrix} \quad (23)$$

$$C_1=-C_2+\frac{Br\,a\,{A_3}^2}{2a-S\,\Pr}+\frac{Br\,a\,{A_4}^2}{2a+S\,\Pr}+\frac{2Br\,A_3(A_1\,a+G)}{a^2-S\,\Pr a}+\frac{2Br\,A_4(A_1\,a+G)}{a^2+S\,\Pr a}$$
$$+\frac{2Br\,a\,A_4 G}{a^2+S\,\Pr a}\left(\frac{2a+S\,\Pr}{a^2+S\,\Pr a}\right)+\frac{2Br\,a\,A_3 G}{a^2-S\,\Pr a}\left(\frac{2a-S\,\Pr}{a^2-S\,\Pr a}\right) \quad (24)$$

4. Result and discussion

Figure 1 demonstrates the effect of couple stresses on the entropy generation in a porous channel with uniform wall temperature. The flow is subjected to slip at its porous walls. Fig. 2 gives an idea about the effect of couple stresses on the velocity profile. It is observed that an increase in the couple stress parameter has decreasing effect on the velocity profile due to rise in the fluid viscosity. Also the velocity is maximum at the center of the channel, i.e., the resistance to flow rises and subsequently the velocity falls in the suction wall. The velocity profile for different values of injection/suction parameter is given in Fig. 3. It is seen that the velocity field increases in middle of the channel with an increase in injection/suction parameter. Fig.4 shows that the effect of upper wall Navier slip parameter on the velocity profile. If we slightly increase the value of upper wall Navier slip parameter that increases velocity in the upper wall and coincide in lower wall. At the same time Fig.5 shows that the effect of lower wall Navier slip parameter on velocity profile. It is noted that an increase the value of lower wall Navier slip parameter that decreases the velocity in the lower wall. In Fig.6 the effect of couple stress parameter on the temperature profile is presented. As a result, increase the couple stress parameter is enhance the temperature profile. While an increase in the suction/injection parameter is studied to break the flow symmetry viewing that the flow is skewed towards the plate with suction as shown in Figs 7.

Figs 8-9 demonstrate the effect of upper and lower wall parameter Navier slip respectively. It is observed that the temperature profile decreases when upper wall navier slip parameter increase. At the same time increase the value of lower wall Navier slip parameter that increase temperature profile.Fig.10 is sketched to see the variation of brinkman number on temperature profile. It is observed that an increase the brinkman number to leads to increases temperature profile. In Figure 11, it is demonstrated that an increase in the fluid Prandtl number decreases the fluid temperature due to rise in the fluid viscosity, because a higher Prandtl number fluid has relatively lower thermal conductivity which reduces conduction and there by decreases the variation. From Figs.12 and 13, it is noted that an increase the value of couple stress parameter and Brinkman number increases the entropy generation respectively. Fig 14 is a representation of the effect of injection/suction parameter. It is found that an increase the value of injection/suction parameter is to reduce entropy generation. Figs15 and 16 denote the effect of upper and lower Navier slip parameters respectively. It is seen that the entropy generation reduce in the injection wall and increase in the suction wall. Figure 17, it is observed that an increase in the couple stress parameter shows that fluid viscosity controls the irreversibility ratio at the centre line of the channel at the same time as heat transfer dominates the irreversibility ratio at the walls. Moreover, an increase in the suction/injection parameter is noted to reduce the heat transfer at the wall with injection and fluid viscosity dominates at the channel centre as seen in Figure 18. However, heat transfer is observed to be dominating the irreversibility ratio at the wall with suction.

Also, in Figures 19–21 heat transfer increases the irreversibility ratio at the walls due to rise in the kinetic energy of the fluid particles while in the centre of the channel fluid viscosity dominates. Finally, in Figure 22, an increase in Prandtl number decreases dominance of heat transfer on the irreversibility ratio at the wall with injection while heat transfer dominates the irreversibility ratio at the suction wall. From Fig.23,it is observed the effect of upper and lower Navier slip parameter on entropy generation dominates the centerline of the channel and increases in the wall.Fig.24 is representation of uniform effect of prandlt number and injection/suction parameter on entropy generation,In Fig.25,it is seen that an increase the value of injection/suction parameter and Brinkman number increases the value of entropy generation. From Fig.26 demonstrate the combined effect of Navier slip parameters on bejan number. Fig.27 is noted the ratio of Brinkman number and thermal parameter uniformly increase the value of bejan number.At last from the Fig.28 it is observed that an increase the value of Prandlt number increases the value of bejan number with uniform effect of Brinkman number.

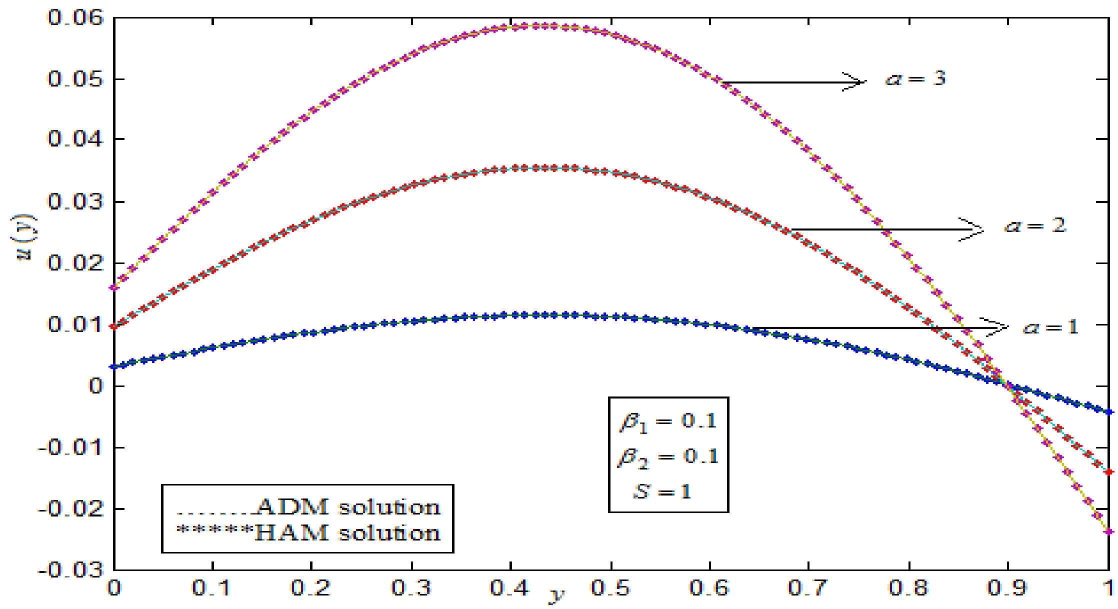

Fig. 2: Dimensionless velocity $u(y)$ versus dimensionless distance y for various values of couple stress parameter (a) and in some fixed values of other dimensionless parameters.

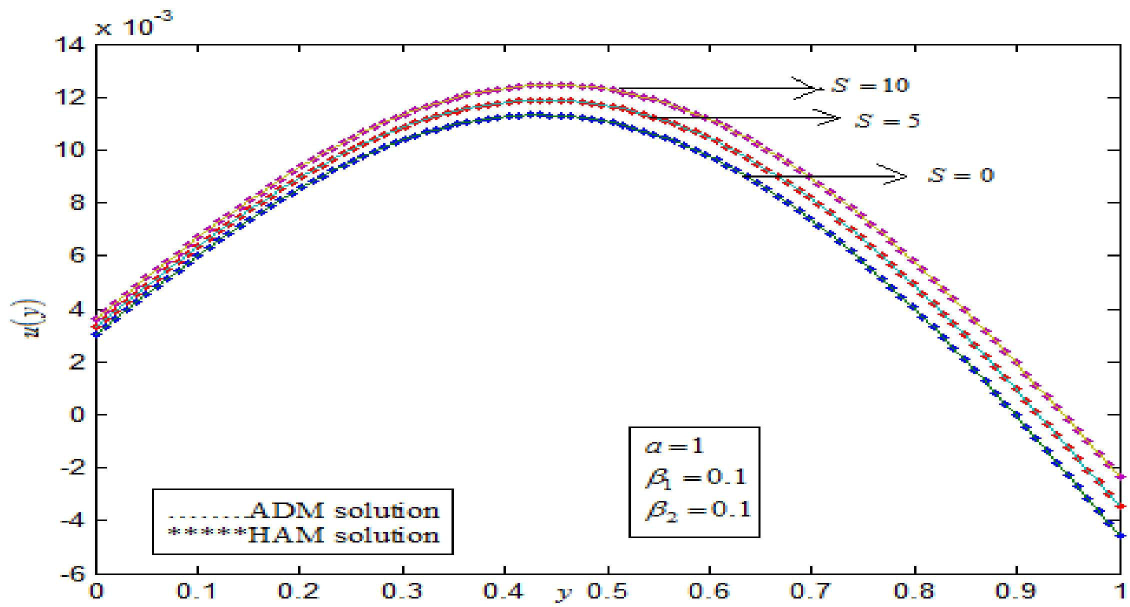

Fig.3: Dimensionless velocity $u(y)$ versus dimensionless distance y for various values of injection/suction parameter (S) and in some fixed values of other dimensionless parameters.

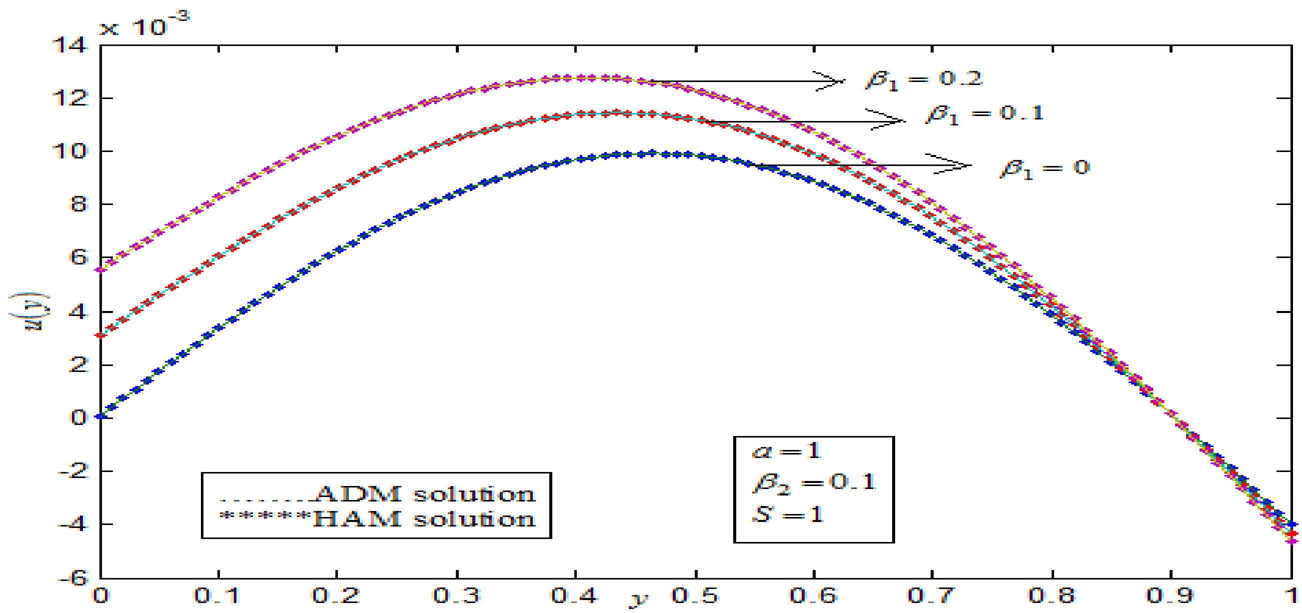

Fig.4: Dimensionless velocity $u(y)$ versus dimensionless distance y for various values of upper wall Navier slip parameter (β_1) and in some fixed values of other dimensionless parameters.

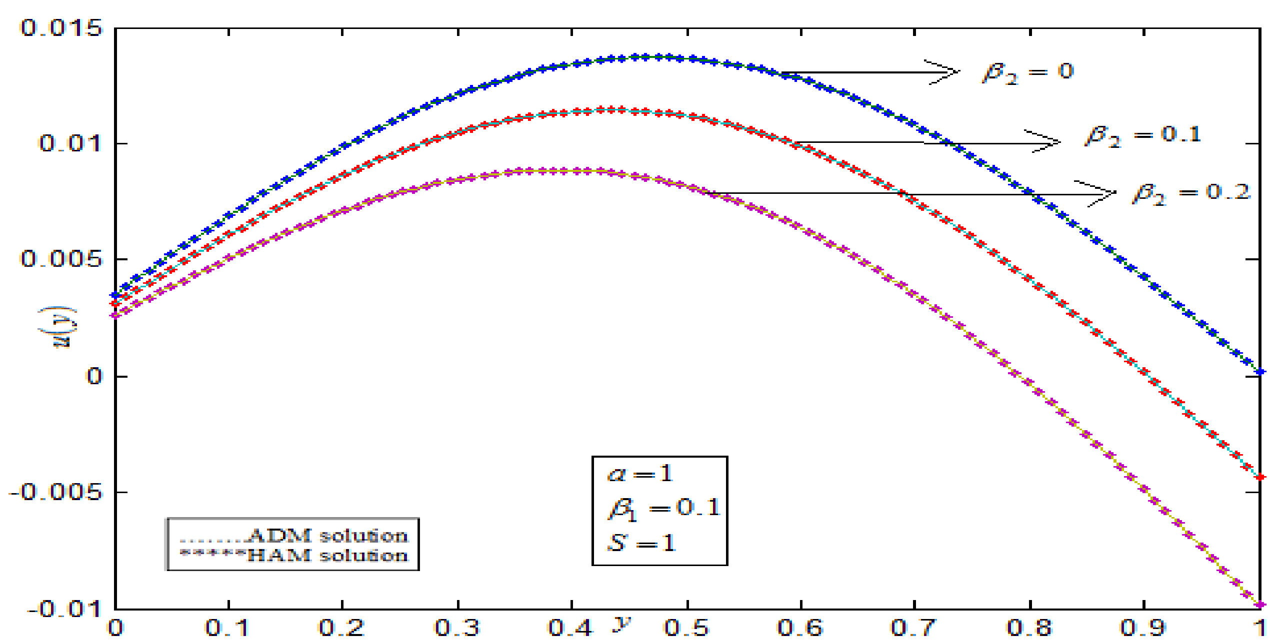

Fig.5: Dimensionless velocity $u(y)$ versus dimensionless distance y for various values of lower wall Navier slip parameter (β_2) and in some fixed values of other dimensionless parameters.

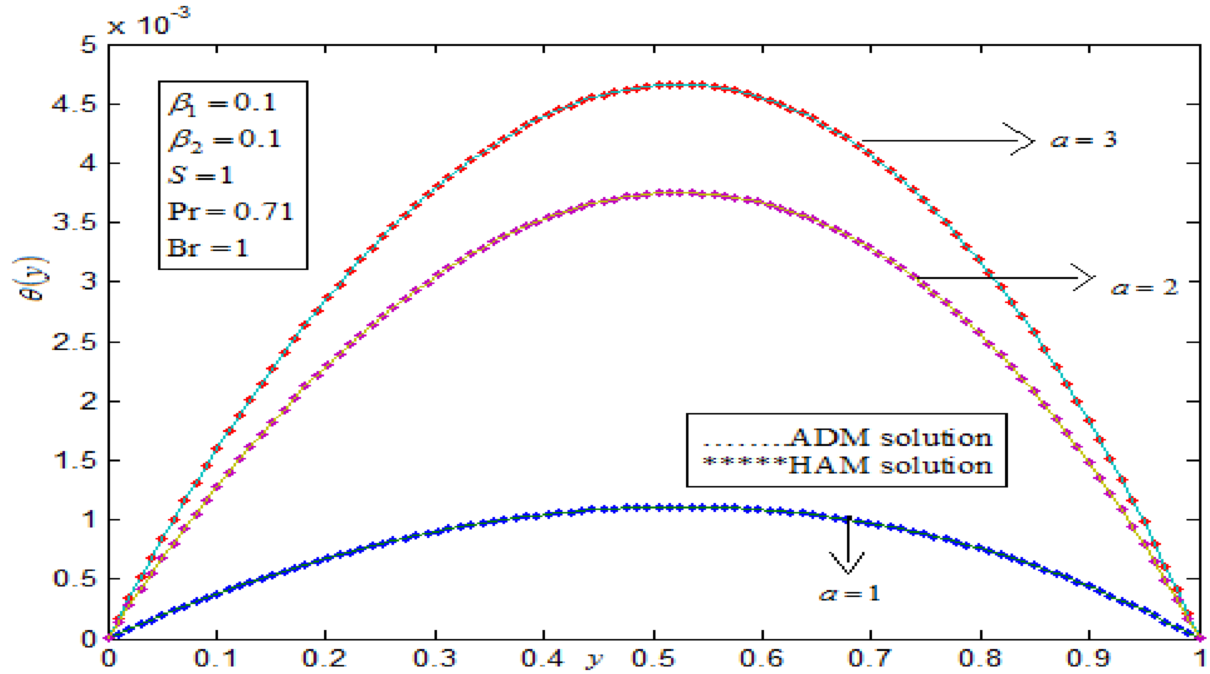

Fig. 6: Dimensionless temperature $\theta(y)$ versus dimensionless distance y for various values of couple stress parameter (a) and in some fixed values of other dimensionless parameters.

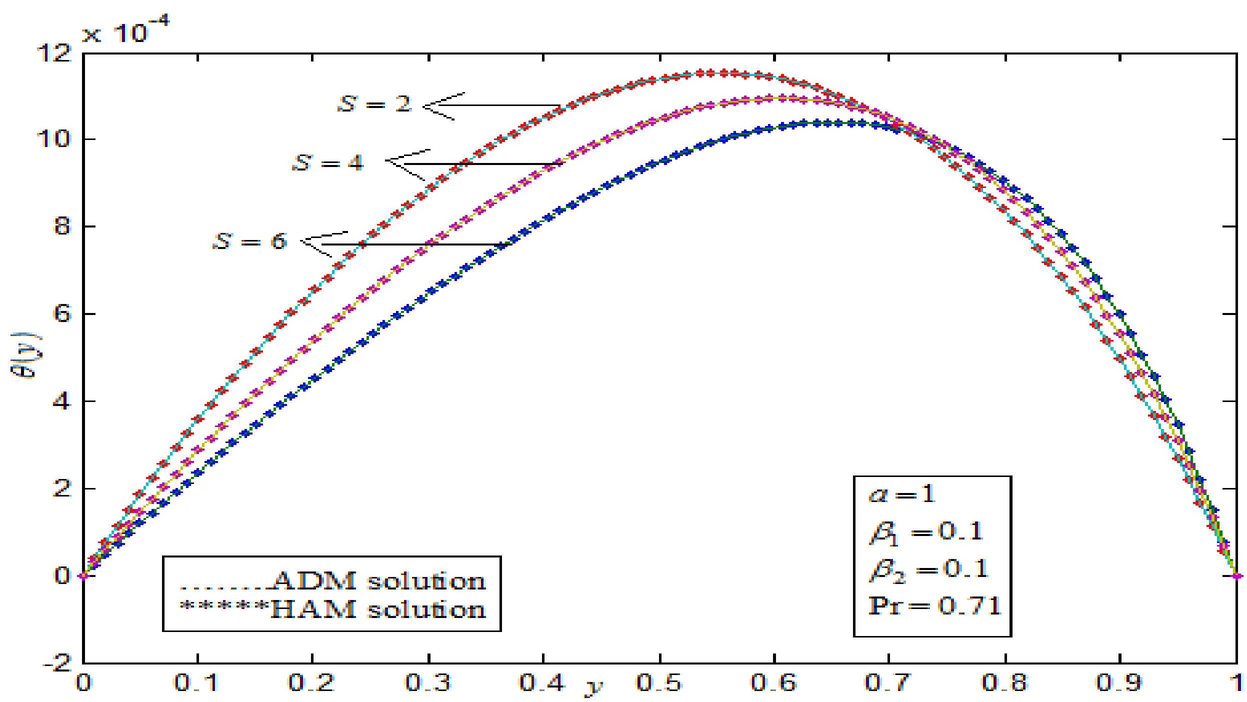

Fig. 7: Dimensionless temperature $\theta(y)$ versus dimensionless distance y for various values of injection/suction parameter (S) and in some fixed values of other dimensionless parameters.

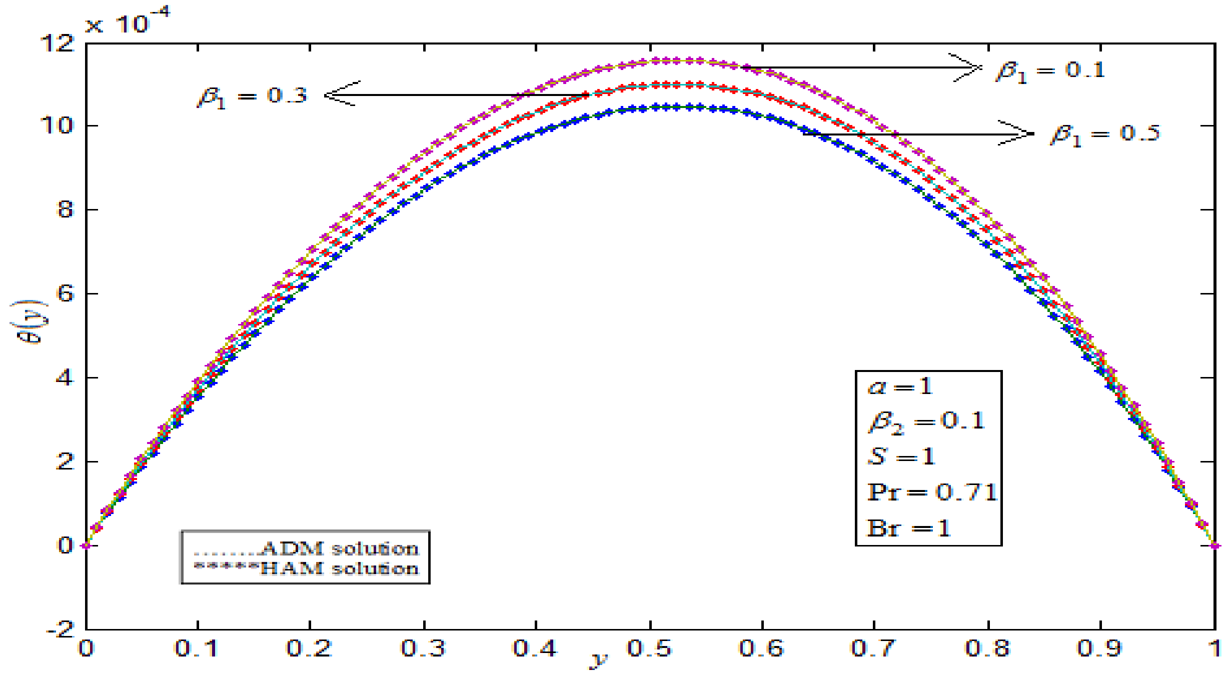

Fig.8: Dimensionless temperature $\theta(y)$ versus dimensionless distance y for various values of upper wall Navier slip parameter (β_1) and in some fixed values of other dimensionless parameter.

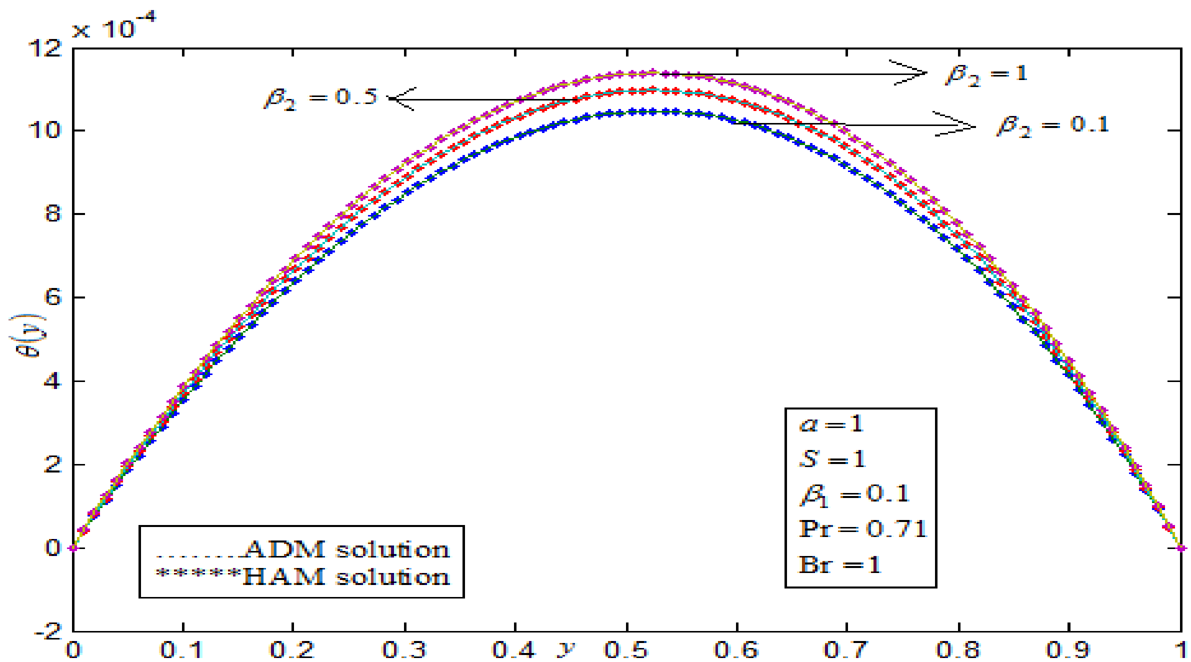

Fig.9: Dimensionless temperature $\theta(y)$ versus dimensionless distance y for various values of lower wall Navier slip parameter (β_2) and in some fixed values of other dimensionless parameters.

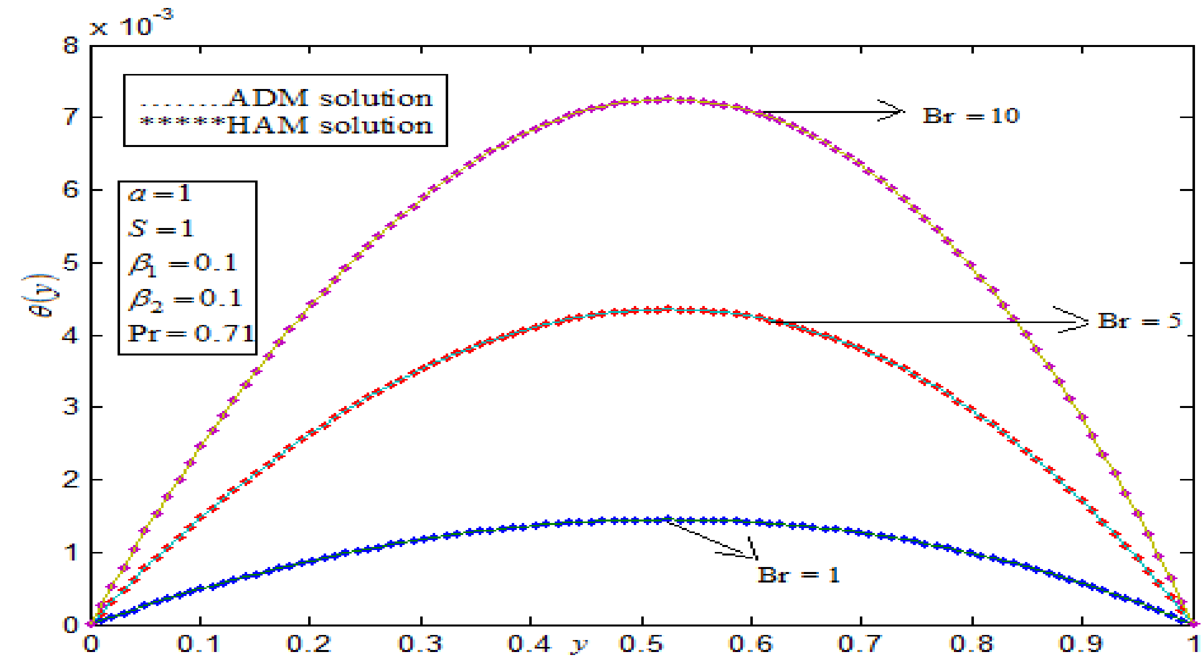

Fig.10: Dimensionless temperature $\theta(y)$ versus dimensionless distance y for various values of Brinkman number (Br) and in some fixed values of other dimensionless parameters.

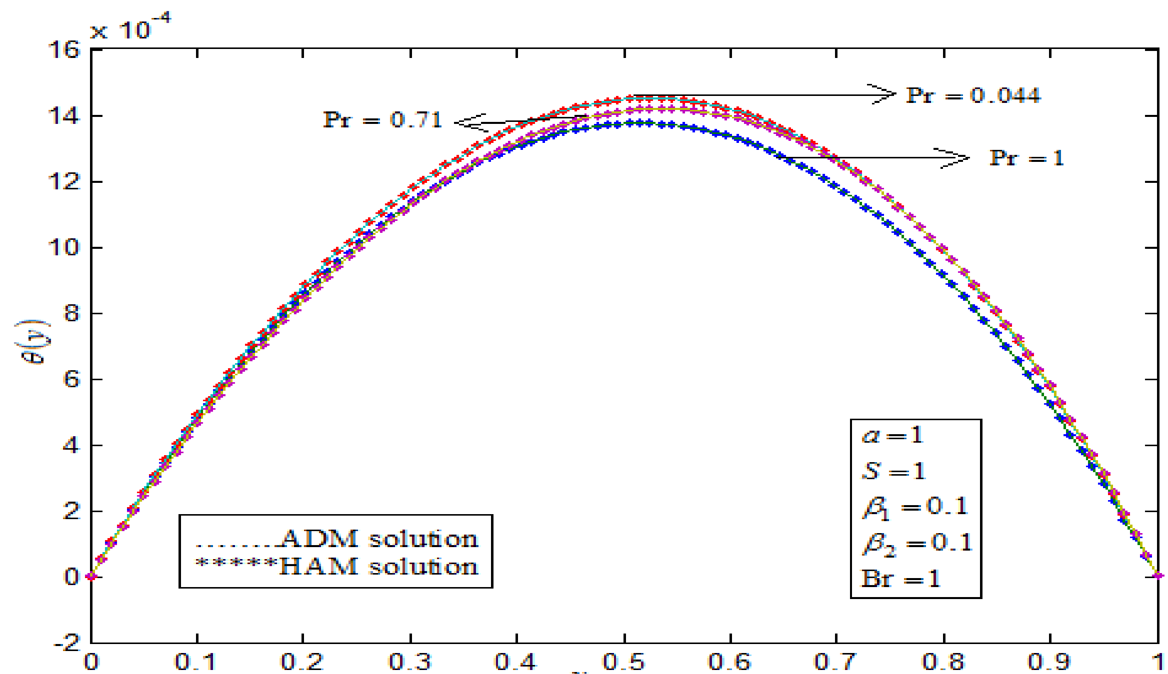

Fig.11: Dimensionless temperature $\theta(y)$ versus dimensionless distance y for various values of Prandtl number($\Pr$) and in some fixed values of other dimensionless parameters.

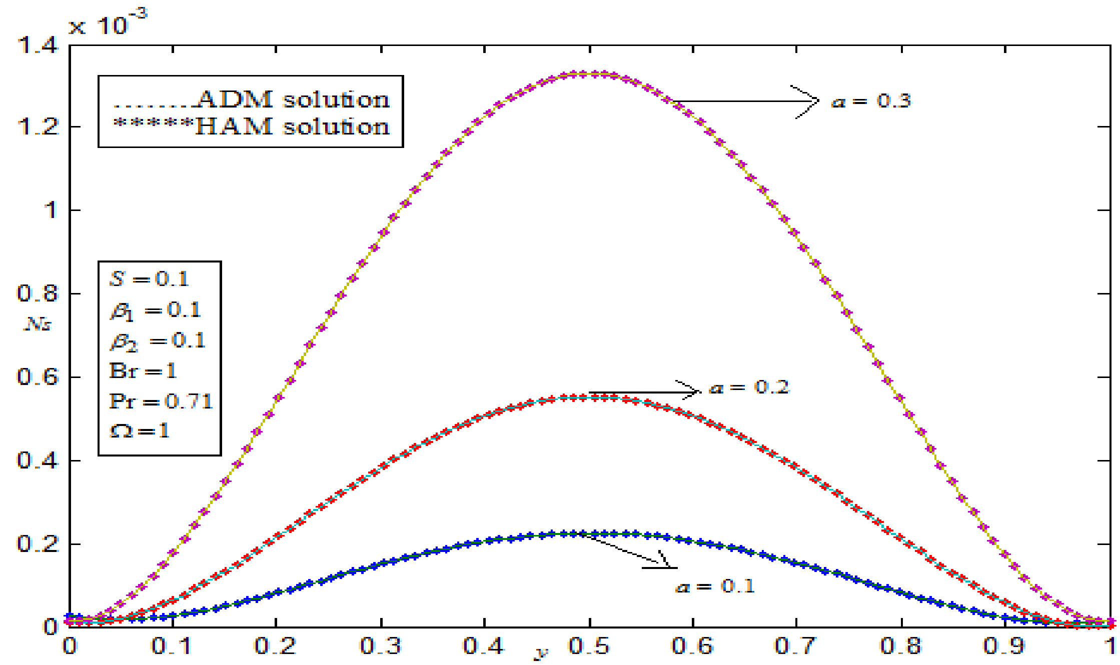

Fig. 12: Entropy generation Ns versus dimensionless distance y for various values of couple stress parameter (a) and in some fixed values of other dimensionless parameters.

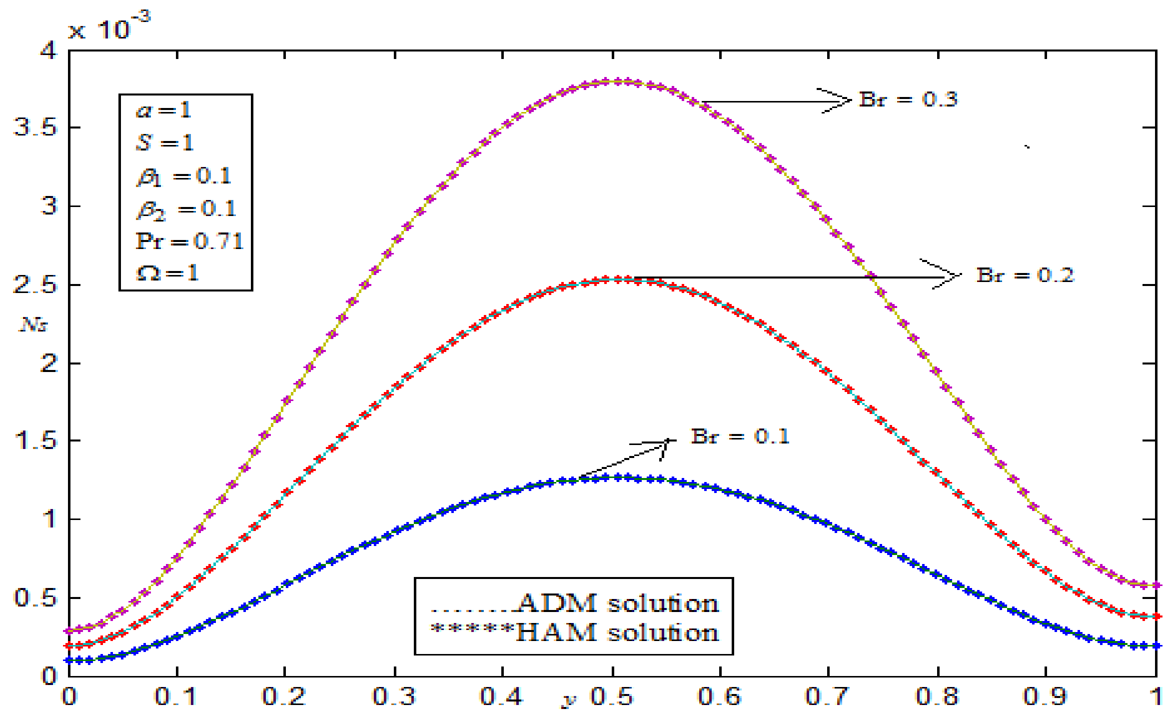

Fig. 13: Entropy generation Ns versus dimensionless distance y for various values of Brinkman number (Br) and in some fixed values of other dimensionless parameters.

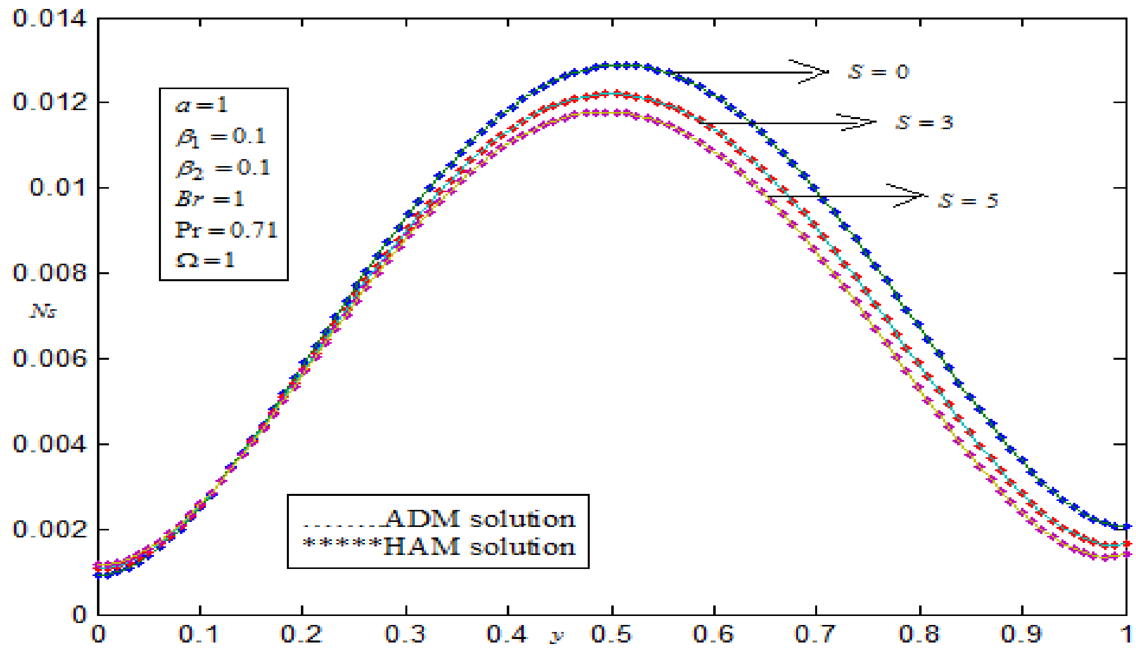

Fig. 14: Entropy generation Ns versus dimensionless distance y for various values of injection/suction parameter (S) and in some fixed values of other dimensionless parameters.

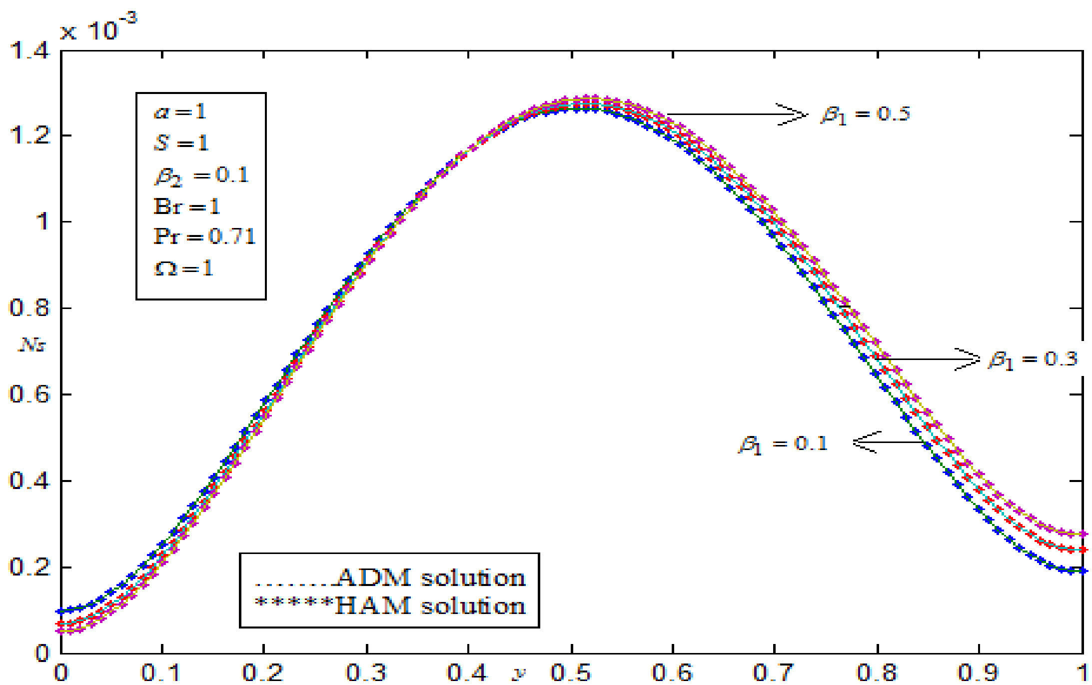

Fig. 15: Entropy generation Ns versus dimensionless distance y for various values of upper wall Navier slip parameter (β_1) and in some fixed values of other dimensionless parameters.

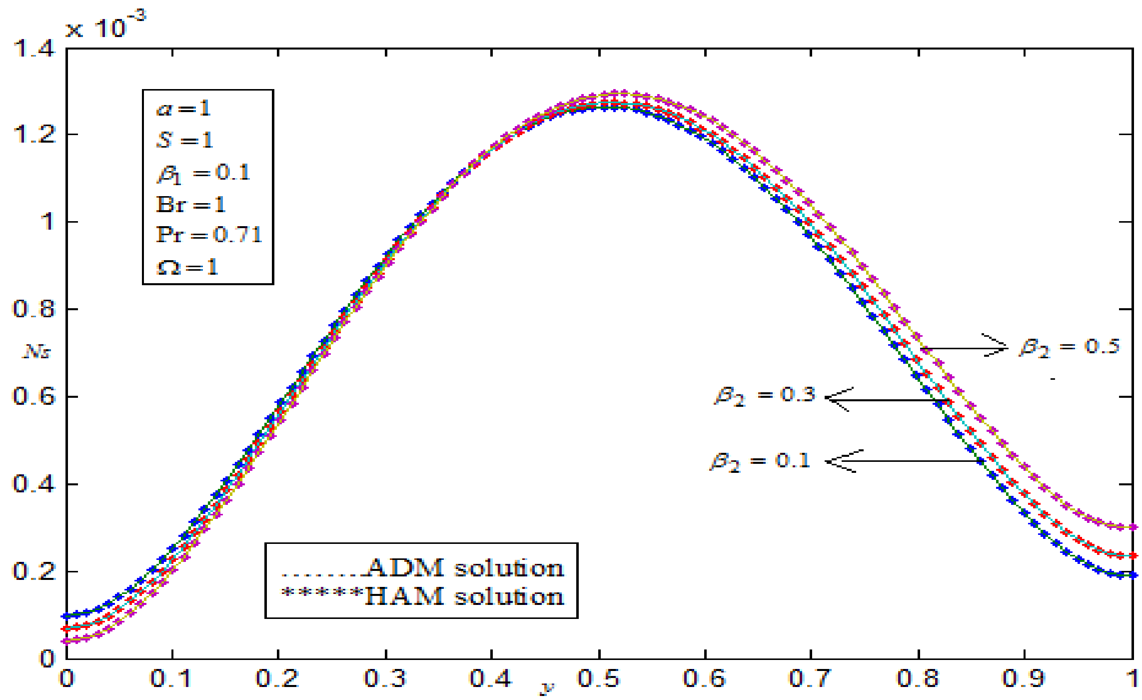

Fig. 16: Entropy generation Ns versus dimensionless distance y for various values of lower wall Navier slip parameter (β_2) and in some fixed values of other dimensionless parameters.

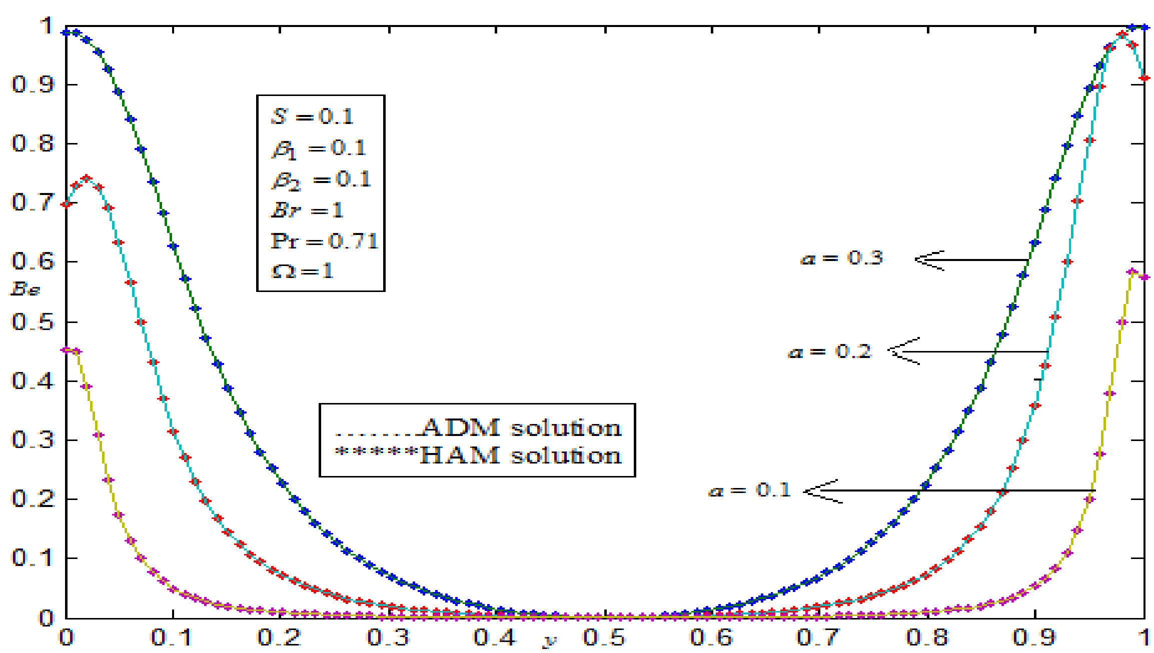

Fig. 17: Bejan number Be versus dimensionless distance y for various values of couple stress parameter (a) and in some fixed values of other dimensionless parameters.

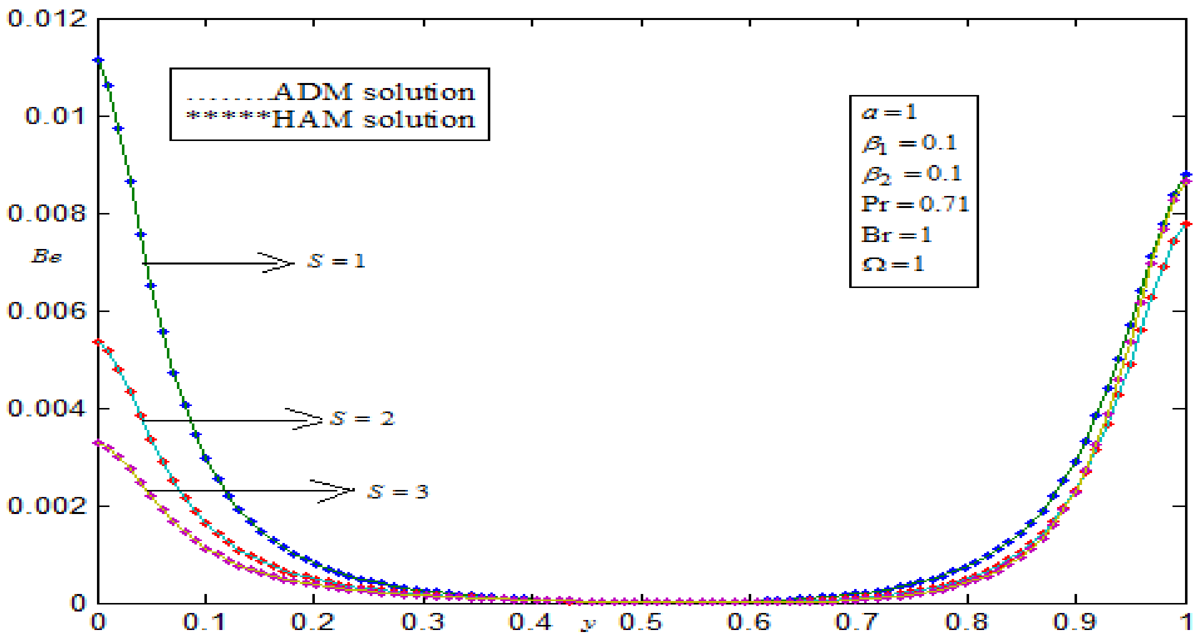

Fig. 18: Bejan number Be versus dimensionless distance y for various values of injection/suction parameter (S) and in some fixed values of other dimensionless parameters.

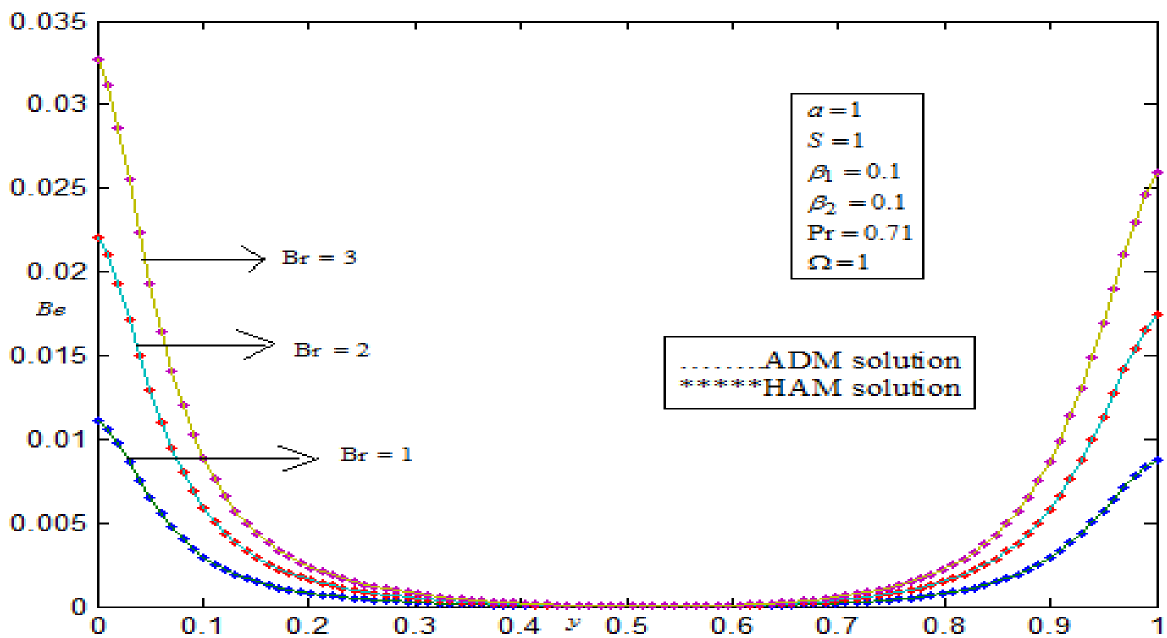

Fig. 19: Bejan number Be versus dimensionless distance y for various values of Brinkman number (Br) and in some fixed values of other dimensionless parameters.

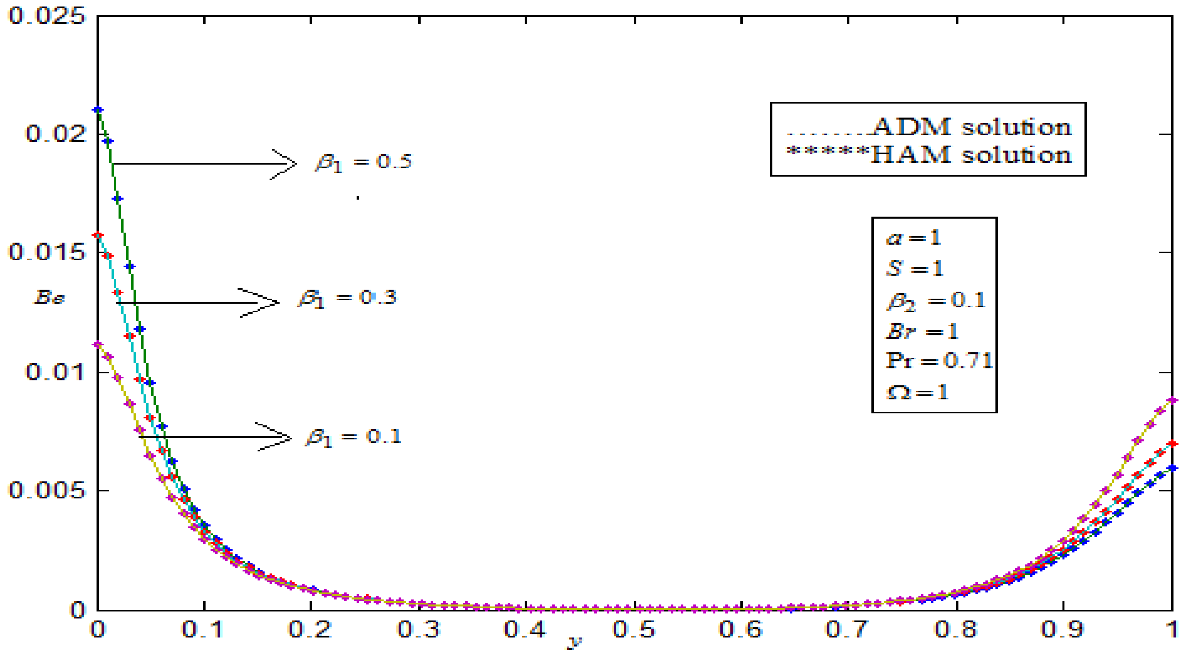

Fig. 20: Bejan number Be versus dimensionless distance y for various values of upper Navier slip parameter (β_1) and in some fixed values of other dimensionless parameters.

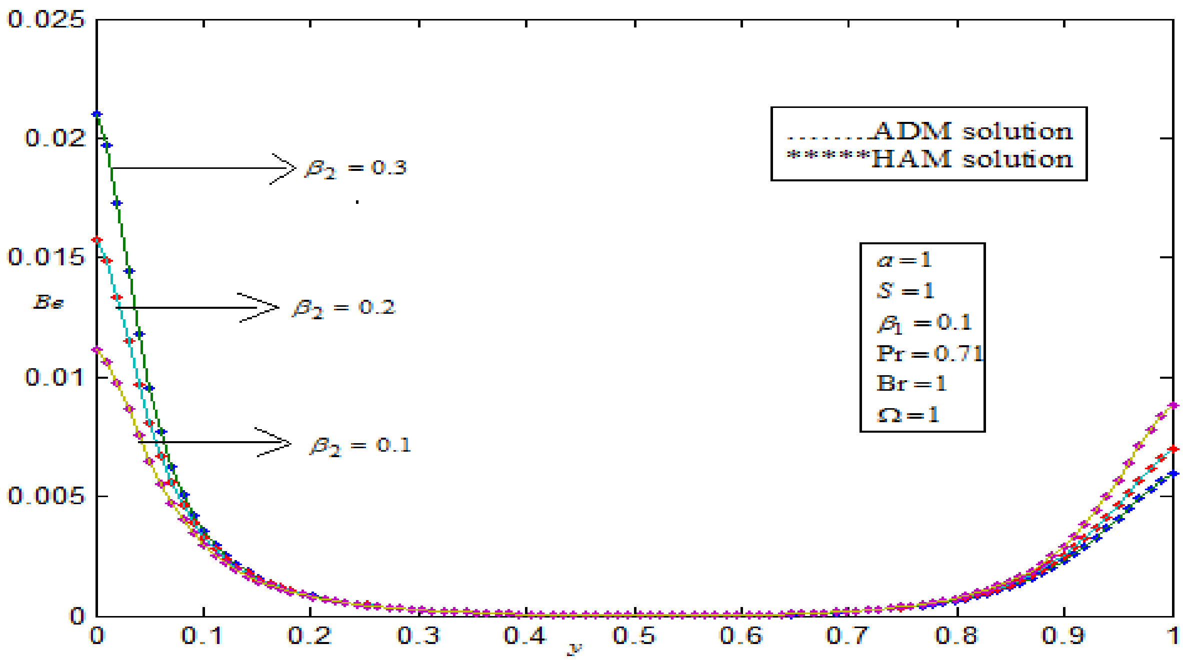

Fig. 21: Bejan number Be versus dimensionless distance y for various values of lower Navier slip parameter (β_2) and in some fixed values of other dimensionless parameters.

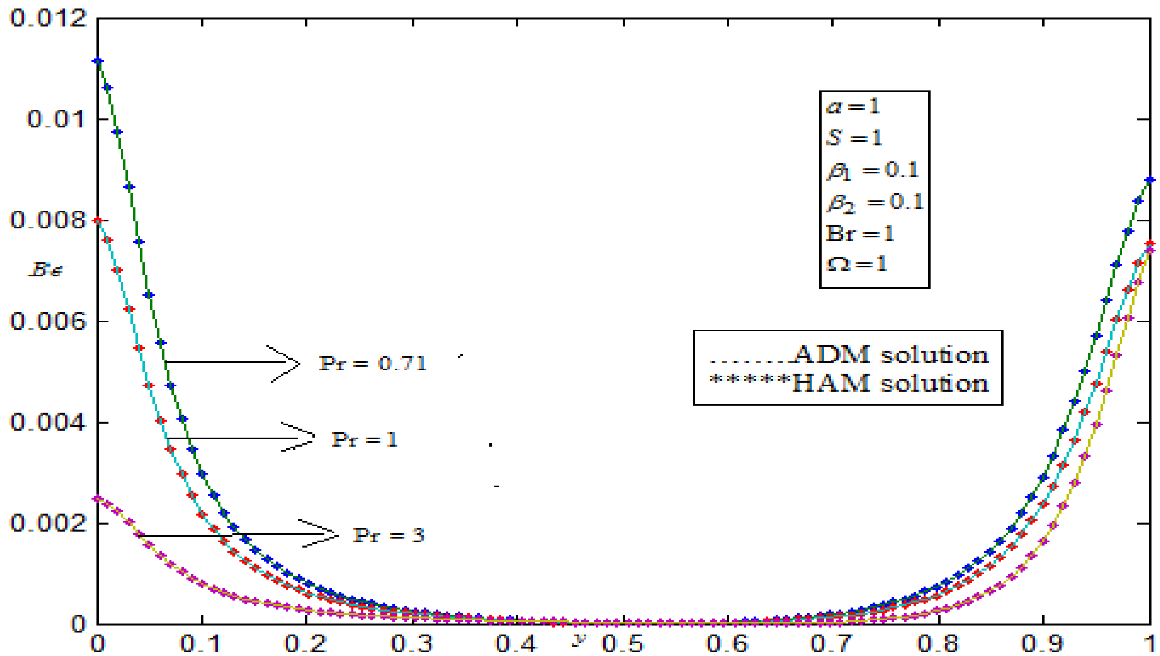

Fig. 22: Bejan number Be versus dimensionless distance y for various values of Prandlt number $(\Pr)$ and in some fixed values of other dimensionless parameters.

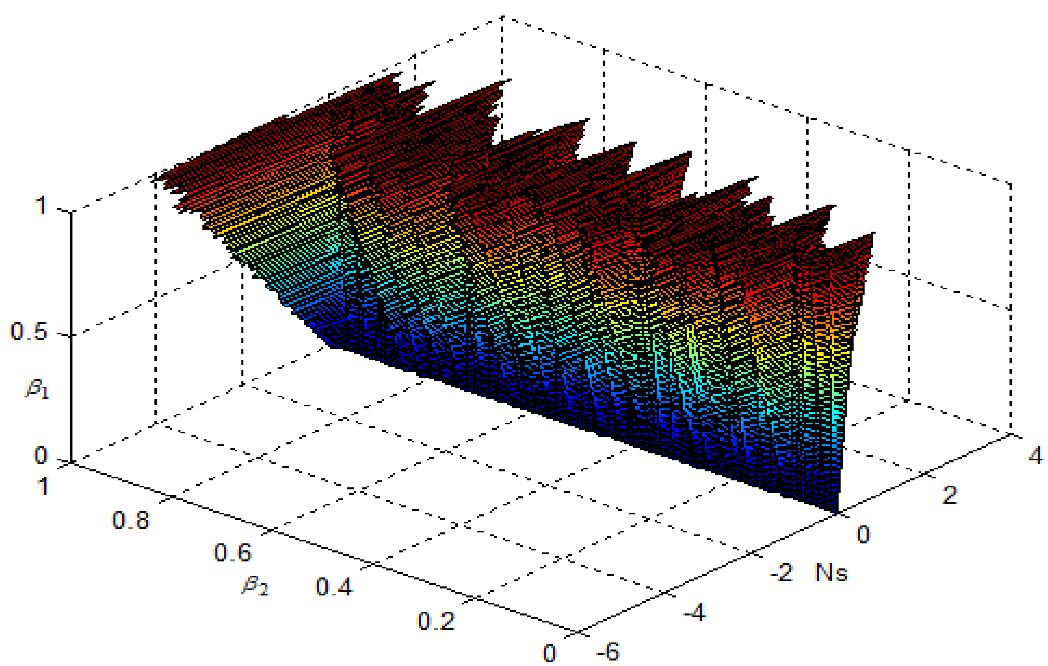

Fig. 23: Effect of upper and lower Navier slip parameter on entropy generation Ns .

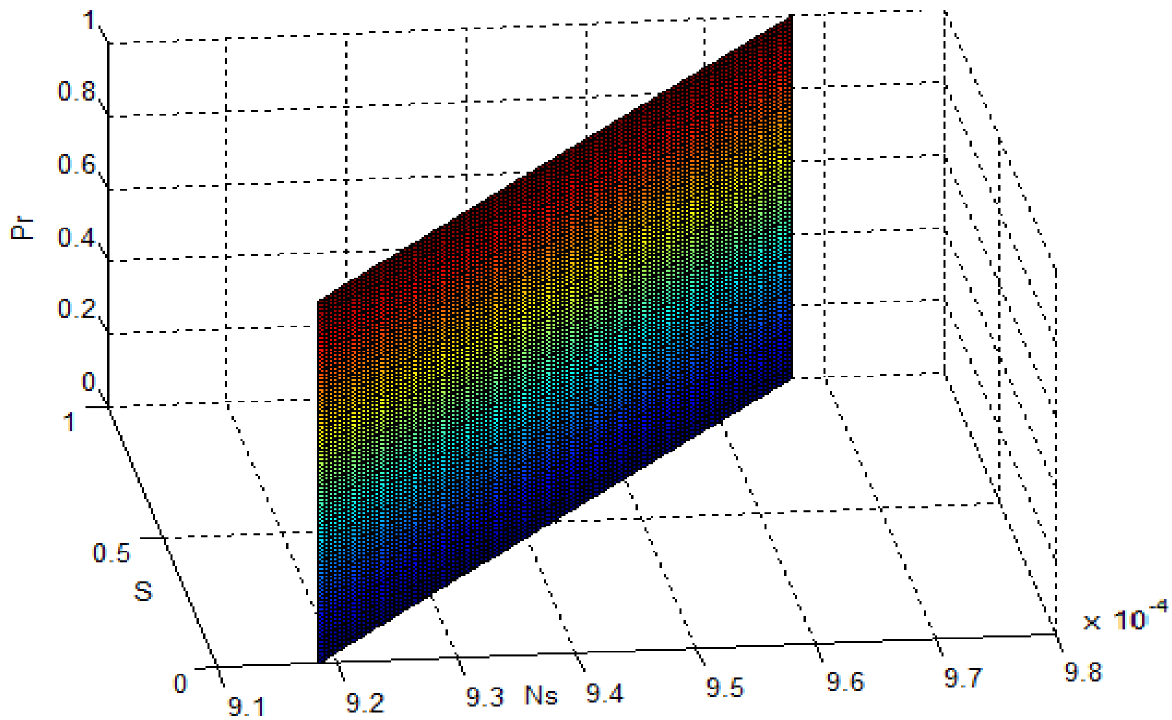

Fig. 24: Effect of Prandlt number and injection/suction parameter on entropy generation Ns .

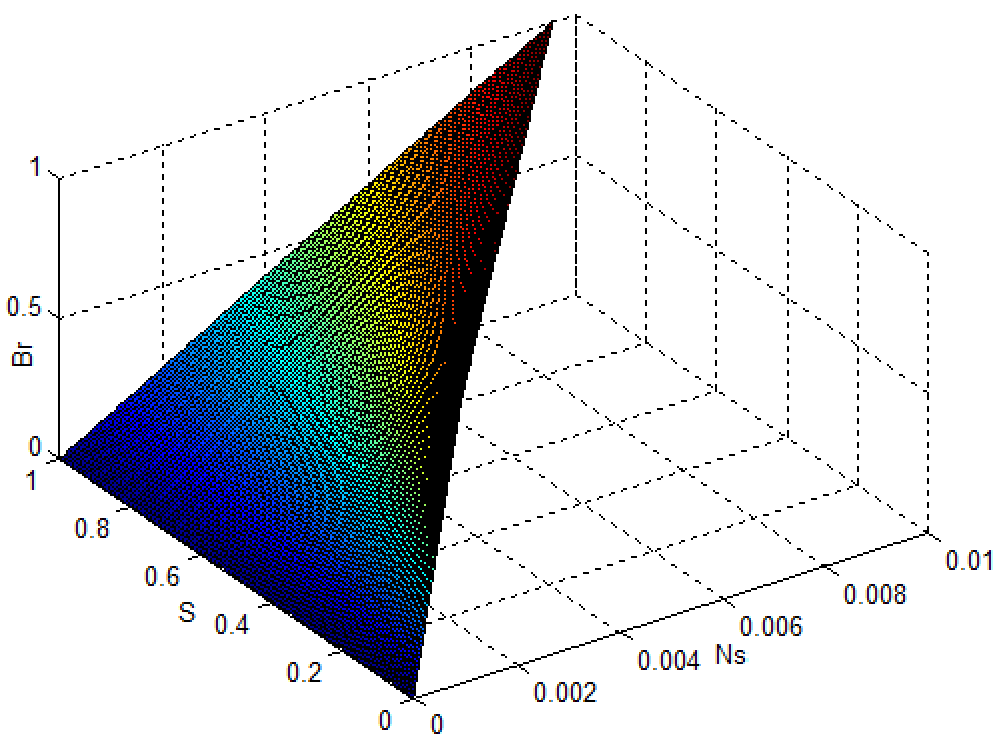

Fig. 25: Effect of Brinkman number and injection/suction parameter on entropy generation Ns .

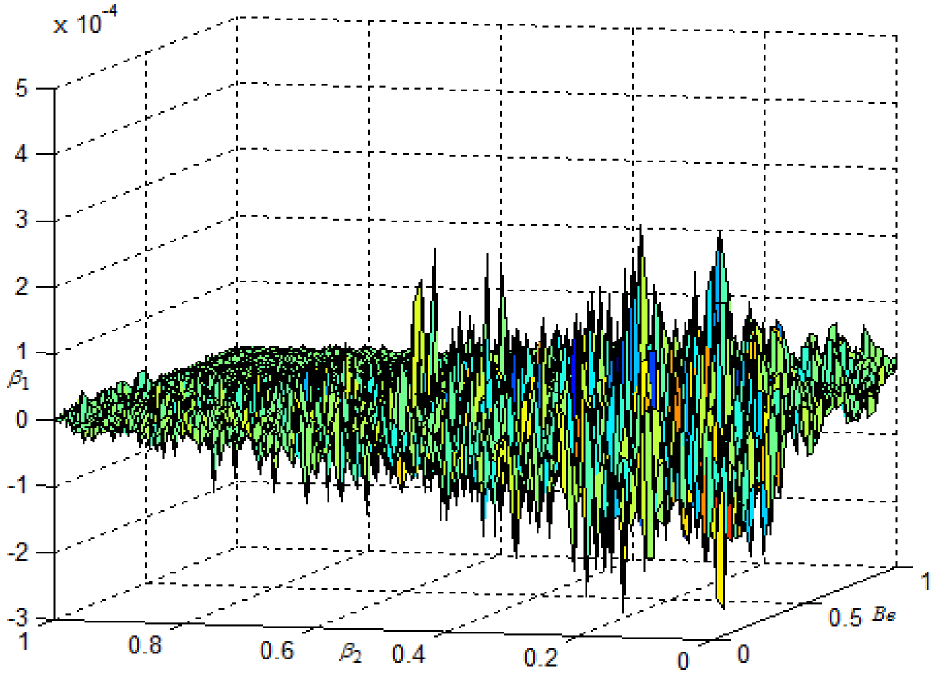

Fig. 26: Effect of upper and lower Navier slip parameter on Bejan number *Be* .

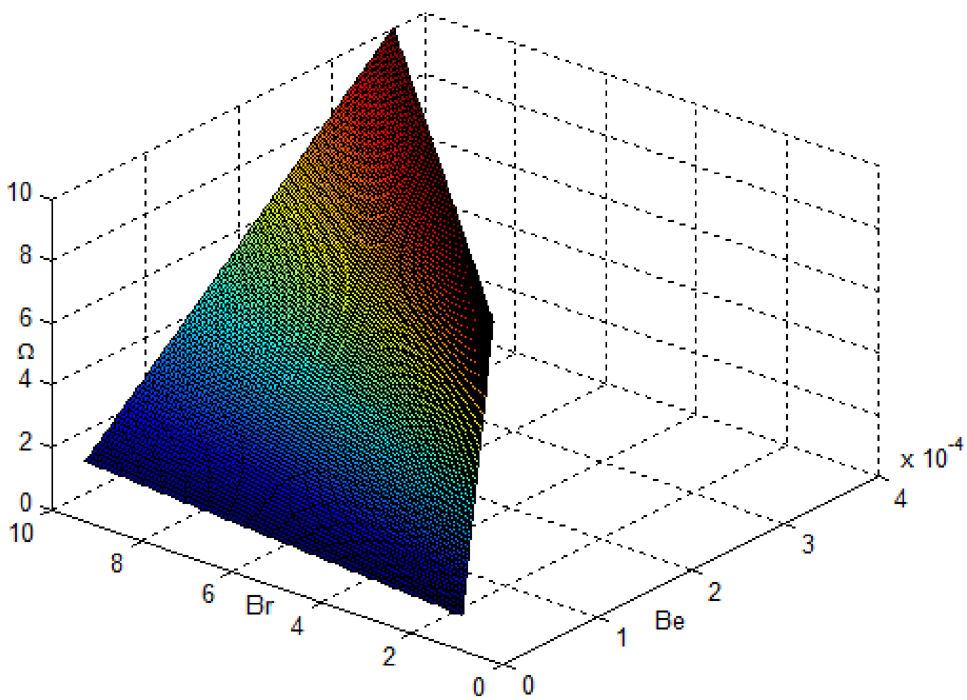

Fig. 27: Effect of Brinkman number and thermal parameter on Bejan number *Be* .

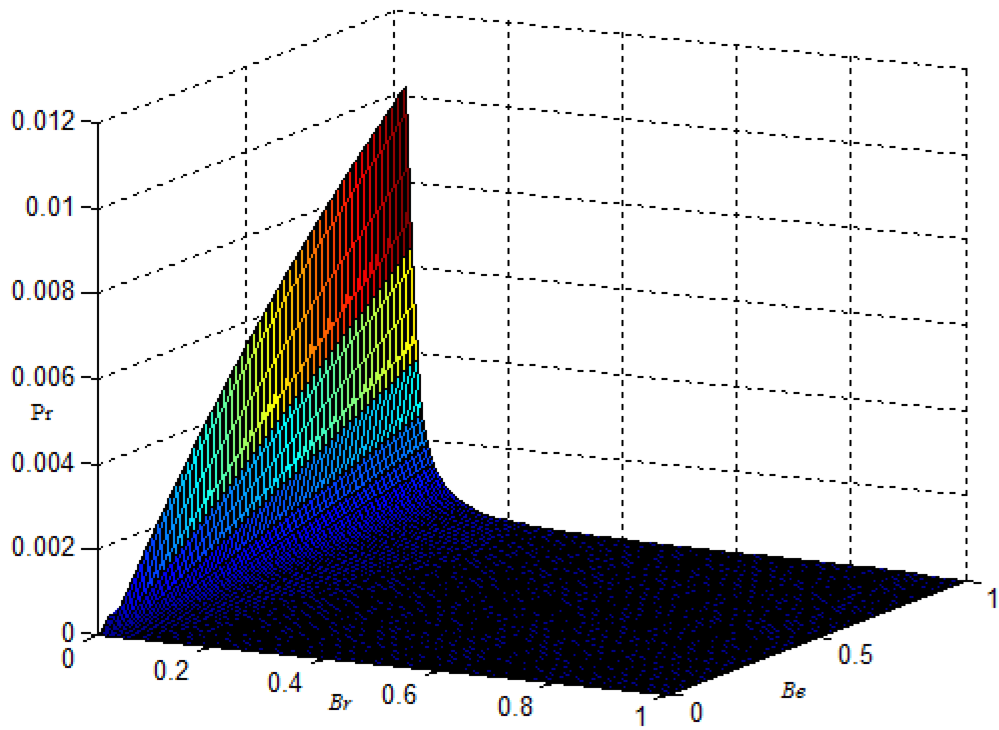

Fig. 28: Effect of Brinkman and Prandlt numbers on Bejan number Be .

5. CONCLUSION

The current study has examined a entropy generation in couple stress fluid flowing steadily through a porous channel with slip at the isothermal walls .The Navier slip model is employed at both walls. HAM is utilized to solve the system of ordinary differential equations. A good agreement can be seen between the results of this study and the results of previously published data with the aid of Matlab software. Analytical solutions in the form of HAM are obtained to approximate value of velocity and temperature fields. The approximate solutions are used to compute the entropy generation and the irreversibility ratio. The effect of couple stress parameter increase all profile. An increase the value of injection/suction parameter is to reduce entropy generation. About the observation of value for brinkman number increase that increases temperature profile, entropy generation and irreversibility ratio. An increase in the fluid Prandtl number decreases the fluid temperature due to rise in the fluid viscosity. Because a higher Prandtl number fluid has relatively lower thermal conductivity which reduces conduction and there by decreases the variation. It also reduces the irreversibility ratio.

6. References

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Appendix – A:

Approximate analytical expression of the non-linear differential eqns.(7) and (8) using new Homotopy analysis method:

In this Appendix, how we can derive the solutions for the eqns. (7) and (8) using Homotopy analysis method We construct the Homotopy for the eqns. (7) and (8) are as follows:

$$(1-p)\left(\frac{d^4u}{dy^4}-a^2\frac{d^2u}{dy^2}-Ga^2\right)=h\,p\left(\frac{d^4u}{dy^4}-a^2\frac{d^2u}{dy^2}-Ga^2+a^2S\frac{du}{dy}\right) \tag{A.1}$$

$$(1-p)\left(\frac{d^2\theta}{dy^2}-S\Pr\frac{d\theta}{dy}\right)=h\,p\left(\frac{d^2\theta}{dy^2}-S\Pr\frac{d\theta}{dy}+Br\left(\frac{du}{dy}\right)^2+\frac{Br}{a^2}\left(\frac{d^2u}{dy^2}\right)^2\right) \tag{A.2}$$

The approximate analytical solution of the eqn.(A.1) and (A.2) are as follows:

$$u=u_0+pu_1+..... \tag{A.3}$$

$$\theta=\theta_0+p\theta_1+........ \tag{A.4}$$

Substituting the eqn.(A.3) into an eqn.(A.1) and (A.4) into an eqn. (A.2) respectively we get the following results:

$$(1-p)\left(\frac{d^4(u_0+pu_1+.....)}{dy^4}-a^2\frac{d^2(u_0+pu_1+.....)}{dy^2}-Ga^2\right)$$
$$=h\,p\left(\begin{array}{l}\dfrac{d^4(u_0+pu_1+.....)}{dy^4}-a^2\dfrac{d^2(u_0+pu_1+.....)}{dy^2}\\ -Ga^2+a^2S\dfrac{d(u_0+pu_1+.....)}{dy}\end{array}\right) \tag{A.5}$$

$$(1-p)\left(\frac{d^2(\theta_0+p\theta_1+.......)}{dy^2}-S\Pr\frac{d(\theta_0+p\theta_1+.......)}{dy}\right)$$
$$=h\,p\left(\begin{array}{l}\dfrac{d^2(\theta_0+p\theta_1+.......)}{dy^2}-S\Pr\dfrac{d(\theta_0+p\theta_1+.......)}{dy}\\ +Br\left(\dfrac{d(u_0+pu_1+.....)}{dy}\right)^2+\dfrac{Br}{a^2}\left(\dfrac{d^2(u_0+pu_1+.....)}{dy^2}\right)^2\end{array}\right) \tag{A.6}$$

Comparing the coefficients of like powers of p in the eqns. (A.5) and (A.6) we get following results:

$$p^0:\frac{d^4u_0}{dy^4}-a^2\frac{d^2u_0}{dy^2}-Ga^2=0 \tag{A.7}$$

$$p^0:\frac{d^2\theta_0}{dy^2}-S\Pr\frac{d\theta_0}{dy}=0 \tag{A.8}$$

$$p^1:\frac{d^4u_1}{dy^4}-a^2\frac{d^2u_1}{dy^2}-\frac{d^4u_0}{dy^4}+a^2\frac{d^2u_0}{dy^2}-Ga^2=h\left(\frac{d^4u_0}{dy^4}-a^2\frac{d^2u_0}{dy^2}-Ga^2+a^2S\frac{du_0}{dy}\right) \tag{A.9}$$

$$p^1:\frac{d^2\theta_1}{dy^2}-S\Pr\frac{d\theta_1}{dy}-\frac{d^2\theta_0}{dy^2}+S\Pr\frac{d\theta_0}{dy}=h\left(\frac{d^2\theta_0}{dy^2}-S\Pr\frac{d\theta_0}{dy}+Br\left(\frac{du_0}{dy}\right)^2+\frac{Br}{a^2}\left(\frac{d^2u_0}{dy^2}\right)^2\right) \tag{A.10}$$

The initial approximations are as follows:

$$u=\beta_1\frac{du}{dy},\ \frac{d^2u}{dy^2}=0,\quad \theta=0\ \ on\ y=0 \tag{A.11}$$

$$u=\beta_2\frac{du}{dy},\ \frac{d^2u}{dy^2}=0,\quad \theta=0\quad on\ y=1 \tag{A.12}$$

Solving the eqns. (A.7) to (A.10) and using the eqns. (A.11) and (A.12) we obtain the following results:

$$u_0=A_1y+A_2+A_3e^{ay}+A_4e^{-ay}+\frac{y^2G}{2} \tag{A.13}$$

$$\theta_0=0 \tag{A.14}$$

$$u_1=A_5y+A_6+A_7e^{ay}+A_8e^{-ay}+\frac{S\,A_1}{2}y^2-\frac{S\,A_3}{2}ye^{ay}-\frac{S\,A_4}{2}ye^{-ay}-\frac{S\,G\,y^3}{6} \tag{A.15}$$

$$\begin{aligned}\theta_1 = {} & C_1+C_2e^{S\Pr y}+Br\left(A_1^2+\frac{G^2}{a^2}\right)\frac{y}{S\Pr}-\frac{Br\,a\,A_3{}^2e^{2ay}}{2a-S\Pr}-\frac{Br\,a\,A_4{}^2e^{-2ay}}{2a+S\Pr}\\ & -\frac{2Br\,A_3(A_1a+G)e^{ay}}{a^2-S\Pr a}-\frac{2BrA_4(A_1a+G)e^{-ay}}{a^2+S\Pr a}+\frac{Br\,G^2}{S\Pr}\left(\frac{y^3}{3}+\frac{y^2}{S\Pr}+\frac{2y}{S^2\Pr^2}\right)\\ & -\frac{2Br\,a\,A_4Ge^{-ay}}{a^2+S\Pr a}\left(y+\frac{2a+S\Pr}{a^2+S\Pr a}\right)+\frac{2Br\,a\,A_3Ge^{ay}}{a^2-S\Pr a}\left(y-\frac{2a-S\Pr}{a^2-S\Pr a}\right)\\ & -\frac{2BrA_1G}{S\Pr}\left(\frac{y^2}{2}+\frac{y}{S\Pr}\right)\end{aligned} \tag{A.16}$$

According to the HAM, we conclude that

$$u=\lim_{p\to\infty}u(y)=u_0+u_1 \tag{A.17}$$

$$\theta=\lim_{p\to\infty}\theta(y)=\theta_0+\theta_1 \tag{A.18}$$

After putting the eqns. (A.13) and (A.15) into an eqn.(A.17) and the eqns.(A.14)and (A.16)) into an eqn. (A.18), we obtain the solutions in the text eqns. (17) and (18) respectively. Where the constants A_1 to A_8 and C_2 and C_1 are defined in the text eqns.(15) – (24) respectively.

Appendix: B Nomenclature

Symbol	Meaning
u'	Axial velocity
μ	Dynamic viscosity
P'	Fluid pressure
ρ	Fluid density
T'	Fluid temperature
T_0	Initial fluid temperature
T_f	Final fluid temperature
k	Thermal conductivity of the fluid
C_p	Specific heat at constant pressure
v_0	Constant velocity of fluid suction/injection
η	Fluid particle size effect due to couple stresses

E_G	Local volumetric entropy generation rate
$\gamma_{1,2}$	Navier slip coefficients
u	Dimensionless velocity
S	Suction/injection parameter
θ	Dimensionless temperature
a	Couple stress parameter
Pr	Prandtl number
Br	Brinkman number
Ω	Paraameter that measures the temperature difference between the two heat reservoirs
N_s	Dimensionless entropy generation rate
S	Dimensionless pressure gradient
β_1, β_2	Upper and lower Navier slip parameters respectively